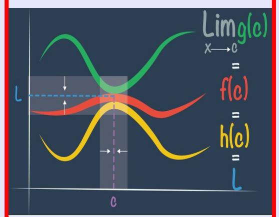
Math 261

Spring 2023

Lecture 40



Feb 19-8:47 AM

Class QZ 11:

Find two positive integers such that the sum of first number (x) and four times the second number (y) is 1000 and their product (xy) is as large as possible.

$\boxed{x + 4y = 1000}$

$x = 1000 - 4y$

xy Maximum

$f(y)$

$(1000-4y)\cdot y = -4y^2 + 1000y$

$f'(y) = -8y + 1000$

$f''(y) = -8 < 0$

C.D. Max

$f'(y)=0$

$-8y + 1000 = 0$

$\boxed{y = 125}$

$x + 4(125) = 1000$

$\boxed{x = 500}$

Final Ans: 500 & 125

Apr 25-9:18 AM

The area of a Circle changes at 100π cm^2/min. $\frac{dA}{dt} = 100\pi$ $cm^2/min.$ $A = \pi r^2$

How fast its $\underbrace{\text{radius changing}}$ when radius is 50 cm?

$\frac{dr}{dt} = ?$ when $r = 50cm$

$A = \pi r^2$

$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}$

$100\pi = \pi \cdot 2 \cdot 50 \cdot \frac{dr}{dt}$

$\to 100\pi = 100\pi \frac{dr}{dt}$

$\frac{dr}{dt} = 1$ cm/min.

Apr 26-8:50 AM

Verify the Conditions of MVT for $f(x) = 3x^2 - 6x + 2$ on $[0, 1]$, then find C on $(0, 1)$ that satisfies the Conclusion of MVT.

$f(x)$ is Polynomial
Cont. & diff. every where.

$f(x) = 3x^2 - 6x + 2$

$f(1) = -1$

$f(0) = 2$

$f'(x) = 6x - 6$

$f'(c) = \frac{f(b) - f(a)}{b - a}$

$6C - 6 = \frac{-1 - 2}{1 - 0}$

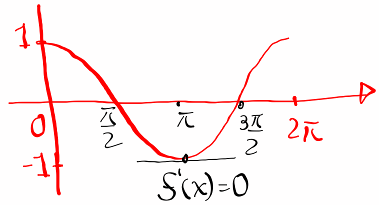
$6C - 6 = -3$

$6C = 3$ $\boxed{C = \frac{1}{2}} \to (0, 1)$

Apr 26-8:54 AM

Verify the conditions of Rolle's theorem for $f(x) = \cos x$ on $[0, 2\pi]$, find all number C which is in the conclusion of Rolle's thrm.

$f(x) = \cos x$



Cont. on $[0, 2\pi]$

Diff. on $(0, 2\pi)$

$f(2\pi) = f(0) = 1$

$f'(c) = 0$

$f(x) = \cos x$

$f'(x) = -\sin x$

$-\sin C = 0$

$\boxed{C = \pi} \rightarrow (0, 2\pi)$

Apr 26-8:59 AM

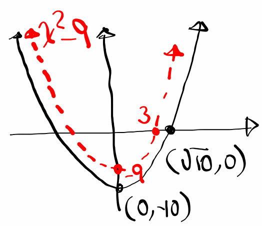
Use Newton's method to estimate $\sqrt{10}$ to 1-decimal place.

Calc. $\sqrt{10} \approx 3.2$

$x = \sqrt{10}$

$x^2 = 10$

$x^2 - 10 = 0$



$f(x) = x^2 - 10$

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 10}{2x_n} = \frac{x_n^2 + 10}{2x_n}$$

$x_1 = 3$

$$x_2 = \frac{3^2 + 10}{2(3)} = \frac{19}{6} = 3.1\overline{6} \approx 3.2$$

$$x_3 = \frac{3.2^2 + 10}{2(3.2)} = \frac{20.24}{6.4} \approx 3.2$$

Apr 26-9:04 AM

Use Calc. method to graph $f(x)=\frac{x^2+1}{x^2}$.

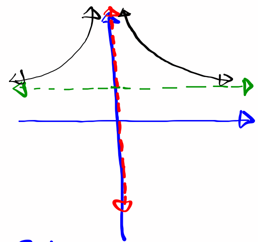
Domain: $x\neq 0 \rightarrow (-\infty,0)\cup(0,\infty)$

Y-Int $\rightarrow$ None
$x=0$ but

X-Int $\rightarrow$ None
$y=0 \rightarrow x^2+1=0$ No Real Soln.

V.A. $x=0$ (Twice)

H.A. $\lim_{x\to\pm\infty} f(x)=1 \rightarrow y=1$



$f(-x)=\frac{(-x)^2+1}{(-x)^2}=\frac{x^2+1}{x^2}=f(x)$

even function $\rightarrow$ Y-axis Sym.

$f(x)=\frac{x^2}{x^2}+\frac{1}{x^2}=1+x^{-2}$

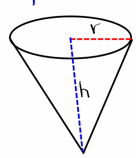
$f'(x)=-2x^{-3}$ $\quad f'(x)=\frac{-2}{x^3}$

$f''(x)=6x^{-4}$ $\quad f''(x)=\frac{6}{x^4}>0 \rightarrow$ C.U.

x	$-\infty$		0		∞
$f'(x)$		+	ϕ	−	
$f''(x)$		+	ϕ	+	
$f(x)$		$\nearrow$	!	$\searrow$	

Apr 26-9:11 AM

A cone-shape paper cup holds $\boxed{27\,cm^3}$ of water. Find height and radius of the cup that will use the least amount of Paper.



Volume of a Circular Cone
$V=\frac{1}{3}\pi r^2 h$
$27=\frac{1}{3}\pi r^2 h$

Open top
Surface area of the Cup
$A=\pi r\sqrt{r^2+h^2}$

$81=\pi r^2 h$

$h=\frac{81}{\pi r^2}$ $\quad h^2=\frac{81^2}{\pi^2 r^4}$ $\quad A=\pi r\cdot\sqrt{r^2+\frac{81^2}{\pi^2 r^4}}$

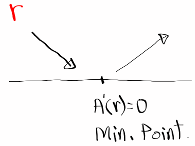
$A=\pi r\cdot\sqrt{\frac{r^2\cdot\pi^2 r^4+81^2}{\pi^2 r^4}}=\pi r\cdot\frac{\sqrt{\pi^2 r^6+81^2}}{\sqrt{\pi^2 r^4}}$

$A(r)=\cancel{\pi r}\cdot\frac{\sqrt{\pi^2 r^6+81^2}}{\cancel{\pi} r^{\cancel{2}}\, r}$

$A(r)=\frac{\sqrt{\pi^2 r^6+81^2}}{r}$

$A'(r)$

$A'(r)=0$



Apr 26-9:20 AM

$f''(x) = 24x^2 + 6x + 4$

$f(2) = 3$

$f(1) = 10$

Find $f(x)$.

$f'(x) = 24 \cdot \frac{x^3}{3} + 6 \cdot \frac{x^2}{2} + 4x + C$

$f'(x) = 8x^3 + 3x^2 + 4x + C$

$f(x) = 8 \cdot \frac{x^4}{4} + 3 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^2}{2} + Cx + D$

$f(x) = 2x^4 + x^3 + 2x^2 + Cx + D$

$f(2) = 3$ $\quad 2 \cdot 2^4 + 2^3 + 2 \cdot 2^2 + 2C + D = 3$

$32 + 8 + 8 + 2C + D = 3$

$\boxed{2C + D = -45}$

$f(1) = 10$ $\quad 2 \cdot 1^4 + 1^3 + 2 \cdot 1^2 + 1C + D = 10$

$2 + 1 + 2 + C + D = 10$

$\boxed{C + D = -5}$

Solve

$$\begin{cases} 2C + D = -45 \\ C + D = -5 \end{cases}$$

Apr 26-9:35 AM

$f'''(x) = \cos x \rightarrow f''(x) = \sin x + C$

$f(0) = 1$

$f'(0) = 2$

$f''(0) = 3$

Find $f(x)$.

$f''(0) = \cancel{\sin 0}^{\,0} + C = 3$

$\boxed{C = 3}$

$f''(x) = \sin x + 3$

$f'(x) = -\cos x + 3x + C$

$f'(0) = -\cancel{\cos 0}^{\,1} + \cancel{3(0)}^{\,0} + C = 2$

$-1 + C = 2$ $\quad \boxed{C = 3}$

$f'(x) = -\cos x + 3x + 3$

$f(x) = -\sin x + 3 \cdot \frac{x^2}{2} + 3x + C$

$f(x) = -\sin x + \frac{3}{2}x^2 + 3x + C$

$f(0) = -\cancel{\sin 0}^{\,0} + \cancel{\frac{3}{2}(0)}^{\,0} + \cancel{3(0)}^{\,0} + C = 1$

$\boxed{C = 1}$

$\boxed{f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1}$

Apr 26-9:43 AM